

Logical Agents

Seung-Hoon Na¹

¹Department of Computer Science
Chonbuk National University

2017.10.10

Wumpus world: The current state of the world

- a square is *breezy*: a neighboring square has a pit
a square is *smelly*: a neighboring square has a wumpus

$$B_{1,1} \quad \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$S_{1,1} \quad \Leftrightarrow (W_{1,2} \vee W_{2,1})$$

...

- There is *at least* one wumpus.

$$W_{1,1} \vee W_{1,2} \vee \dots \vee W_{4,3} \vee W_{4,4}$$

- There is *at most* one wumpus.

$$\neg W_{1,1} \vee \neg W_{1,2}$$

$$\neg W_{1,1} \vee \neg W_{1,3}$$

$$\neg W_{1,1} \vee \neg W_{1,4}$$

...

$$\neg W_{4,3} \vee \neg W_{4,4}$$

Associating propositions with time steps

- A percept should assert something *only about the current time*: E.g. $Stench^t$, $Breeze^t$, $Glitter^t$, etc.
 - Otherwise, suppose that there was no stench at the previous time, i.e., $\neg Stench$ was asserted. Then, if there is currently a stench, we cannot add $Stench$ at the current time, because the new assertion results in a contradiction.
- **Fluent**: refers to an aspect of the world that changes.
- **Atemporal variances**: symbols associated with permanent aspects of the world do not need a time superscript.
- For any time step t and any square $[x, y]$, we assert:

$$L_{x,y}^t \Leftarrow (Breeze^t \Leftrightarrow B_{x,y})$$
$$L_{x,y}^t \Leftarrow (Stench^t \Leftrightarrow S_{x,y})$$

- Transition models on fluents: Fluents change as the result of actions taken by the agent.
- **Effect axioms:** Specify the outcome of an action at the next time step.

$$L_{1,1}^0 \wedge FacingEast^0 \wedge Forward^0 \Leftarrow (L_{2,1}^1 \wedge \neg L_{1,1}^1)$$

...

- **Frame problem:** need to represent a long list of facts that are not changed by an action
- Suppose that the agent move *Forward* at time 0. Given effect axioms, we have $L_{2,1}^1$. Thus, $ASK(KB, L_{2,1}^1) = true$.
- However, $ASK(KB, HaveArrow^1) = false$: The agent cannot prove it still has the arrow now doesn't have it.
- Effect axioms only fail to state what remains *unchanged* as the result of an action.

- **Frame axioms:** As one possible solution to the frame problem, we can add frame axioms that explicitly asserts all the propositions that remain the same.

$$\text{Forward}^t \Leftarrow (\text{HaveArrow}^t \Leftrightarrow \text{HaveArrow}^{t+1})$$

$$\text{Forward}^t \Leftarrow (\text{WumpusAlive}^t \Leftrightarrow \text{WumpusAlive}^{t+1})$$

...

- **Representational frame problem:** With m actions and n fluents, the set of frame axioms is $\mathcal{O}(mn)$, being largely inefficient.
- Need to use **Locality:** Each action typically changes no more than some small number k of fluents, thus requiring to define $\mathcal{O}(mk)$, rather than $\mathcal{O}(mn)$.
- **Inferential frame problem:** The problem of projecting forward the results of t step plan of action in time $\mathcal{O}(kt)$ rather than $\mathcal{O}(nt)$ - The need to reason explicitly about things that don't change.

Successor-state axiom

- **Successor-state axiom:** Axioms about *fluents*. For each fluent F , a successor-state axiom defines F^{t+1} in terms of fluents at time t and the actions that may have occurred at time t , with the following schema:

$$F^{t+1} \Leftrightarrow \text{ActionCauses}F^t \vee (F^t \wedge \neg \text{ActionCausesNot}F^t)$$

- For *HaveArrow*

$$\text{HaveArrow}^{t+1} \Leftrightarrow (\text{HaveArrow}^t \wedge \neg \text{Shoot}^t)$$

- For the agent's location

$$\begin{aligned} L_{1,1}^{t+1} &\Leftrightarrow (L_{1,1}^t \wedge (\neg \text{Forward}^t \vee \text{Bump}^{t+1})) \\ &\vee (L_{1,2}^t \wedge (\text{South}^t \wedge \text{Forward}^t)) \\ &\vee (L_{2,1}^t \wedge (\text{West}^t \wedge \text{Forward}^t)) \end{aligned}$$

Wumpus world: Question about the current state of the world

- Given the initial sequence of percepts and actions:

$\neg Stench^0 \wedge \neg Breeze^0 \wedge \neg Glitter^0 \wedge \neg Bump^0 \wedge \neg Scream^0 ; Forward^0$
 $\neg Stench^1 \wedge Breeze^1 \wedge \neg Glitter^1 \wedge \neg Bump^1 \wedge \neg Scream^1 ; TurnRight^1$
 $\neg Stench^2 \wedge Breeze^2 \wedge \neg Glitter^2 \wedge \neg Bump^2 \wedge \neg Scream^2 ; TurnRight^2$
 $\neg Stench^3 \wedge Breeze^3 \wedge \neg Glitter^3 \wedge \neg Bump^3 \wedge \neg Scream^3 ; Forward^3$
 $\neg Stench^4 \wedge \neg Breeze^4 \wedge \neg Glitter^4 \wedge \neg Bump^4 \wedge \neg Scream^4 ; TurnRight^4$
 $\neg Stench^5 \wedge \neg Breeze^5 \wedge \neg Glitter^5 \wedge \neg Bump^5 \wedge \neg Scream^5 ; Forward^5$
 $Stench^6 \wedge \neg Breeze^6 \wedge \neg Glitter^6 \wedge \neg Bump^6 \wedge \neg Scream^6$

- Now, we have $ASK(KB, L_{1,2}^6) = true$, $ASK(KB, W_{1,3}) = true$, $ASK(KB, P_{1,3}) = true$.
- We can also define the additional axiom to check whether a square is OK:

$$OK_{x,y}^t \Leftrightarrow \neg P_{x,y} \wedge \neg \neg (W_{x,y} \wedge WumpusAlive^t)$$

Then, we have $ASK(KB, OK_{2,2}^6) = true$.

Qualification problem

- We need to confirm *all the necessary preconditions* of an action hold for it to have its intended effect.
 - E.g.: the *Forward* action moves the agent ahead unless there is a wall in the way, but there are many other unusual exceptions that could cause the action to fail: the agent might trip and fall, be stricken with a heart attack, be carried away by giant bats, etc.
- Specifying all these exceptions is called the **qualification problem**: No complete solution within logic, raising a designing problem on knowledge base.

Hybrid agent: combine logical inference with problem-solving ability

```
function HYBRID-WUMPUS-AGENT(percept) returns an action
  inputs: percept, a list, [stench,breeze,glitter,bump,scream]
  persistent: KB, a knowledge base, initially the atemporal “wumpus physics”
               t, a counter, initially 0, indicating time
               plan, an action sequence, initially empty

  TELL(KB, MAKE-PERCEPT-SENTENCE(percept, t))
  TELL the KB the temporal “physics” sentences for time t
  safe  $\leftarrow$   $\{[x, y] : \text{ASK}(\text{KB}, \text{OK}_{x,y}^t) = \text{true}\}$ 
  if  $\text{ASK}(\text{KB}, \text{Glitter}^t) = \text{true}$  then
    plan  $\leftarrow$  [Grab] + PLAN-ROUTE(current,  $\{[1,1]\}$ , safe) + [Climb]
  if plan is empty then
    unvisited  $\leftarrow$   $\{[x, y] : \text{ASK}(\text{KB}, \text{L}_{x,y}^{t'}) = \text{false for all } t' \leq t\}$ 
    plan  $\leftarrow$  PLAN-ROUTE(current, unvisited  $\cap$  safe, safe)
  if plan is empty and  $\text{ASK}(\text{KB}, \text{HaveArrow}^t) = \text{true}$  then
    possible_wumpus  $\leftarrow$   $\{[x, y] : \text{ASK}(\text{KB}, \neg W_{x,y}) = \text{false}\}$ 
    plan  $\leftarrow$  PLAN-SHOT(current, possible_wumpus, safe)
  if plan is empty then // no choice but to take a risk
    not_unsafe  $\leftarrow$   $\{[x, y] : \text{ASK}(\text{KB}, \neg \text{OK}_{x,y}^t) = \text{false}\}$ 
    plan  $\leftarrow$  PLAN-ROUTE(current, unvisited  $\cap$  not_unsafe, safe)
  if plan is empty then
    plan  $\leftarrow$  PLAN-ROUTE(current,  $\{[1, 1]\}$ , safe) + [Climb]
  action  $\leftarrow$  POP(plan)
  TELL(KB, MAKE-ACTION-SENTENCE(action, t))
  t  $\leftarrow$  t + 1
  return action
```

Hybrid agent: combine logical inference with problem-solving ability

function PLAN-ROUTE(*current, goals, allowed*) **returns** an action sequence

inputs: *current*, the agent's current position

goals, a set of squares; try to plan a route to one of them

allowed, a set of squares that can form part of the route

problem \leftarrow ROUTE-PROBLEM(*current, goals, allowed*)

return A*-GRAPH-SEARCH(*problem*)

- The weakness in the algorithm of hybrid agent: as time goes by, the computational expense involved in the calls to *ASK* goes up and up. This is because the required inferences have to go back further and further in time and involve more and more proposition symbols.
- **Belief state**: some representation of the set of all possible current states of the world. Here, the belief state is a logical sentence.

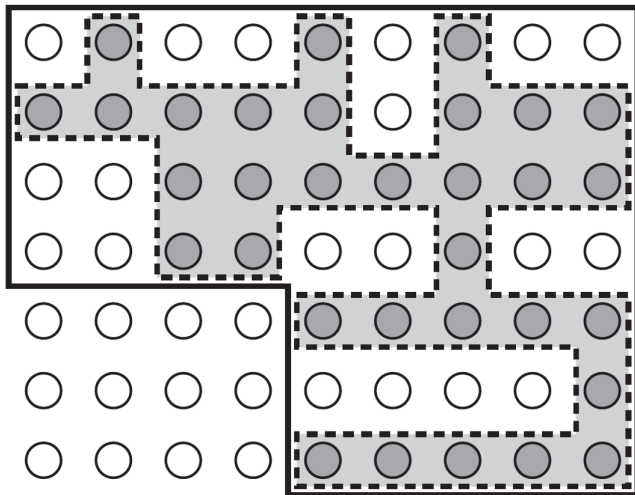
$$WumpusAlive^1 \wedge L_{2,1}^1 \wedge B_{2,1} \wedge (P_{3,1} \vee P_{2,2})$$

- **State estimation**: the process of updating the belief state as new percepts arrive.

- Maintaining an exact belief state as a logical formula: *not tractable*.
 - For n fluent symbols for time t , there are 2^n possible states and 2^{2^n} belief states.
- **Approximate state estimation:** Represent belief states as conjunctions of literals, 1-CNF formulas.
 - The agent program tries to prove X^t and $\neg X^t$ for each symbol X_t , given the belief state at $t - 1$.
 - The conjunction of provable literals becomes the new belief state, and the previous belief state is discarded.

Approximate Logical state estimation

- the 1-CNF belief state acts as a simple outer envelope: The set of possible states represented by the 1-CNF belief state includes all states that are in fact possible given the full percept history.



Making plans by propositional inference

- 1) Construct a sentence that includes.
 - $Init^0$, a collection of assertions about the initial state;
 - $Transition^1, \dots, Transition^t$, the successor-state axioms for all possible actions at each time up to some maximum time t ;
 - the assertion that the goal is achieved at time t :
 $HaveGold^t \wedge ClimbedOut^t$.
- 2) Present the whole sentence to a SAT solver. If the solver finds a satisfying model, then the goal is achievable; if the sentence is unsatisfiable, then the planning problem is impossible.
- 3) Assuming a model is found, extract from the model those variables that represent actions and are assigned *true*. The resulting model is provided as a plan to achieve the goals.

Making plans by propositional inference

```
function SATPLAN(init, transition, goal,  $T_{\max}$ ) returns solution or failure
inputs: init, transition, goal, constitute a description of the problem
          $T_{\max}$ , an upper limit for plan length

for  $t = 0$  to  $T_{\max}$  do
    cnf  $\leftarrow$  TRANSLATE-TO-SAT(init, transition, goal,  $t$ )
    model  $\leftarrow$  SAT-SOLVER(cnf)
    if model is not null then
        return EXTRACT-SOLUTION(model)
return failure
```


SATPLAN: Entailment vs. Satisfiability

- Suppose that $L_{1,1}^0$, we didn't tell the agent that it can't be in two places at once. Then, $L_{2,1}^0$ is unknown
 - For entailment, the unknown literal $L_{2,1}^0$ *cannot* be used in a proof
 - For satisfiability, the unknown literal $L_{2,1}^0$ *can* be set to whatever value helps to make the goal true.
- Thus, *SATPLAN*: a good debugging tool for KBs because it reveals places where knowledge is missing.
 - E.g.: we can fix the knowledge base by asserting that, at each time step, the agent is in exactly one location.

SATPLAN: On satisfiability issue - Additional axioms

- 1. *SATPLAN* finds models with *impossible actions*, such as shooting with no arrow.
 - **Precondition axioms:** stating that an action occurrence requires the preconditions to be satisfied

$$\text{Shoot}^t \Leftarrow \text{HaveArrow}_t \quad (1)$$

- 2. *SATPLAN* finds the models with *multiple simultaneous actions*.
 - **Action exclusion axioms:** every pair of actions A_i^t and A_j^t , we add the axiom

$$\neg A_i^t \vee \neg A_j^t \quad (2)$$