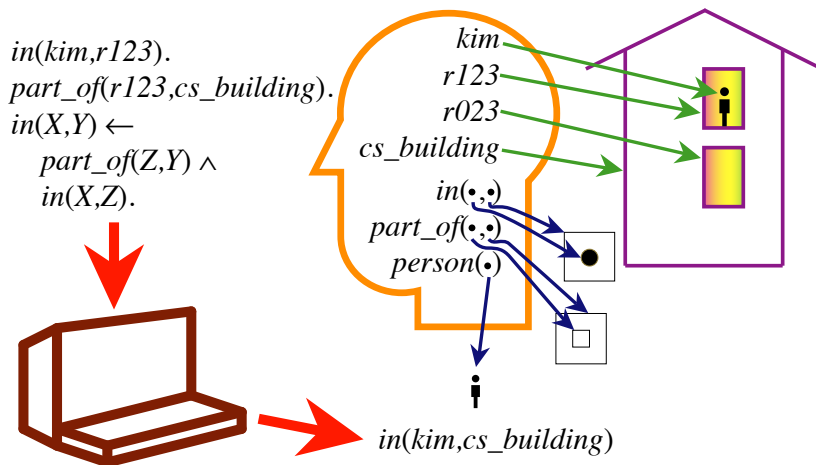


Individuals and Relations

- It is useful to view the world as consisting of individuals (objects, things) and relations among individuals.
- Often features are made from relations among individuals and functions of individuals.
- Reasoning in terms of individuals and relationships can be simpler than reasoning in terms of features, if we can express general knowledge that covers all individuals.
- Sometimes we may know some individual exists, but not which one.
- Sometimes there are infinitely many individuals we want to refer to (e.g., set of all integers, or the set of all stacks of blocks).

Role of Semantics in Automated Reasoning



Features of Automated Reasoning

- Users can have meanings for symbols in their head.
- The computer doesn't need to know these meanings to derive logical consequence.
- Users can interpret any answers according to their meaning.

Decision-theoretic Planning

- flat or modular or hierarchical
- explicit states or features or individuals and relations
- static or finite stage or indefinite stage or infinite stage
- fully observable or partially observable
- deterministic or stochastic dynamics
- goals or complex preferences
- single agent or multiple agents
- knowledge is given or knowledge is learned
- perfect rationality or bounded rationality

Representational Assumptions of Datalog

- An agent's knowledge can be usefully described in terms of *individuals* and *relations* among individuals.
- An agent's knowledge base consists of *definite* and *positive* statements.
- The environment is *static*.
- There are only a finite number of individuals of interest in the domain. Each individual can be given a unique name.

⇒ Datalog

- A **variable** starts with upper-case letter.
- A **constant** starts with lower-case letter or is a sequence of digits (numeral).
- A **predicate symbol** starts with lower-case letter.
- A **term** is either a variable or a constant.
- An **atomic symbol** (atom) is of the form p or $p(t_1, \dots, t_n)$ where p is a predicate symbol and t_i are terms.

Syntax of Datalog (cont)

- A **definite clause** is either an atomic symbol (a fact) or of the form:

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1 \wedge \dots \wedge b_m}_{\text{body}}$$

where a and b_i are atomic symbols.

- **query** is of the form $?b_1 \wedge \dots \wedge b_m$.
- **knowledge base** is a set of definite clauses.

$in(kim, R) \leftarrow$
 $teaches(kim, cs322) \wedge$
 $in(cs322, R).$

$grandfather(william, X) \leftarrow$
 $father(william, Y) \wedge$
 $parent(Y, X).$

$slithy(foves) \leftarrow$
 $mimsy \wedge borogroves \wedge$
 $outgrabe(mome, Raths).$

A **semantics** specifies the meaning of sentences in the language.

An **interpretation** specifies:

- what objects (individuals) are in the world
- the correspondence between symbols in the computer and objects & relations in world
 - ▶ constants denote individuals
 - ▶ predicate symbols denote relations

An **interpretation** is a triple $I = \langle D, \phi, \pi \rangle$, where

- D , the **domain**, is a nonempty set. Elements of D are **individuals**.
- ϕ is a mapping that assigns to each constant an element of D . Constant c **denotes** individual $\phi(c)$.
- π is a mapping that assigns to each n -ary predicate symbol a relation: a function from D^n into $\{TRUE, FALSE\}$.

Example Interpretation

Constants: *phone*, *pencil*, *telephone*.

Predicate Symbol: *noisy* (unary), *left_of* (binary).

- $D = \{\langle \text{✂} \rangle, \langle \text{☎} \rangle, \langle \text{✎} \rangle\}$.
- $\phi(\textit{phone}) = \langle \text{☎} \rangle$, $\phi(\textit{pencil}) = \langle \text{✎} \rangle$, $\phi(\textit{telephone}) = \langle \text{☎} \rangle$.

- $\pi(\textit{noisy})$:

$\langle \text{✂} \rangle$	FALSE	$\langle \text{☎} \rangle$	TRUE	$\langle \text{✎} \rangle$	FALSE
----------------------------	-------	----------------------------	------	----------------------------	-------

$\pi(\textit{left_of})$:

$\langle \text{✂}, \text{✂} \rangle$	FALSE	$\langle \text{✂}, \text{☎} \rangle$	TRUE	$\langle \text{✂}, \text{✎} \rangle$	TRUE
$\langle \text{☎}, \text{✂} \rangle$	FALSE	$\langle \text{☎}, \text{☎} \rangle$	FALSE	$\langle \text{☎}, \text{✎} \rangle$	TRUE
$\langle \text{✎}, \text{✂} \rangle$	FALSE	$\langle \text{✎}, \text{☎} \rangle$	FALSE	$\langle \text{✎}, \text{✎} \rangle$	FALSE

Important points to note

- The domain D can contain real objects. (e.g., a person, a room, a course). D can't necessarily be stored in a computer.
- $\pi(p)$ specifies whether the relation denoted by the n -ary predicate symbol p is true or false for each n -tuple of individuals.
- If predicate symbol p has no arguments, then $\pi(p)$ is either *TRUE* or *FALSE*.

Truth in an interpretation

A constant c **denotes in** I the individual $\phi(c)$.

Ground (variable-free) atom $p(t_1, \dots, t_n)$ is

- **true in interpretation** I if $\pi(p)(\langle\phi(t_1), \dots, \phi(t_n)\rangle) = \text{TRUE}$ in interpretation I and
- **false** otherwise.

Ground clause $h \leftarrow b_1 \wedge \dots \wedge b_m$ is **false in interpretation** I if h is false in I and each b_i is true in I , and is **true in interpretation** I otherwise.

Example Truths

In the interpretation given before, which of following are true?

noisy(phone)

noisy(telephone)

noisy(pencil)

left_of(phone, pencil)

left_of(phone, telephone)

noisy(phone) ← left_of(phone, telephone)

noisy(pencil) ← left_of(phone, telephone)

noisy(pencil) ← left_of(phone, pencil)

noisy(phone) ← noisy(telephone) ∧ noisy(pencil)

Example Truths

In the interpretation given before, which of following are true?

<i>noisy(phone)</i>	true
<i>noisy(telephone)</i>	true
<i>noisy(pencil)</i>	false
<i>left_of(phone, pencil)</i>	true
<i>left_of(phone, telephone)</i>	false
<i>noisy(phone) ← left_of(phone, telephone)</i>	true
<i>noisy(pencil) ← left_of(phone, telephone)</i>	true
<i>noisy(pencil) ← left_of(phone, pencil)</i>	false
<i>noisy(phone) ← noisy(telephone) ∧ noisy(pencil)</i>	true

Models and logical consequences (recall)

- A knowledge base, KB , is true in interpretation I if and only if every clause in KB is true in I .
- A **model** of a set of clauses is an interpretation in which all the clauses are true.
- If KB is a set of clauses and g is a conjunction of atoms, g is a **logical consequence** of KB , written $KB \models g$, if g is true in every model of KB .
- That is, $KB \models g$ if there is no interpretation in which KB is true and g is false.

User's view of Semantics

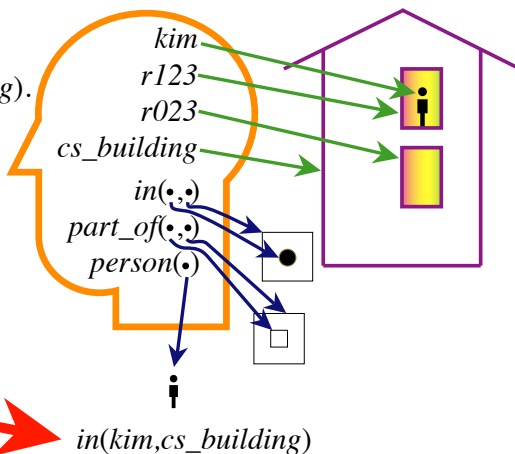
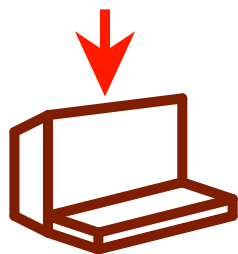
1. Choose a task domain: **intended interpretation**.
2. Associate constants with individuals you want to name.
3. For each relation you want to represent, associate a predicate symbol in the language.
4. Tell the system clauses that are true in the intended interpretation: **axiomatizing the domain**.
5. Ask questions about the intended interpretation.
6. If $KB \models g$, then g must be true in the intended interpretation.

Computer's view of semantics

- The computer doesn't have access to the intended interpretation.
- All it knows is the knowledge base.
- The computer can determine if a formula is a logical consequence of KB.
- If $KB \models g$ then g must be true in the intended interpretation.
- If $KB \not\models g$ then there is a model of KB in which g is false. This could be the intended interpretation.

Role of Semantics in an RRS

$in(kim,r123).$
 $part_of(r123,cs_building).$
 $in(X,Y) \leftarrow$
 $part_of(Z,Y) \wedge$
 $in(X,Z).$



- Variables are **universally quantified** in the scope of a clause.
- A **variable assignment** is a function from variables into the domain.
- Given an interpretation and a variable assignment, each term denotes an individual and each clause is either true or false.
- A clause containing variables is true in an interpretation if it is true **for all** variable assignments.

A **query** is a way to ask if a body is a logical consequence of the knowledge base:

$$?b_1 \wedge \dots \wedge b_m.$$

An **answer** is either

- an instance of the query that is a logical consequence of the knowledge base KB , or
- **no** if no instance is a logical consequence of KB .

Example Queries

$$KB = \begin{cases} in(kim, r123). \\ part_of(r123, cs_building). \\ in(X, Y) \leftarrow part_of(Z, Y) \wedge in(X, Z). \end{cases}$$

Query

Answer

?part_of(r123, B).

Example Queries

$$KB = \begin{cases} in(kim, r123). \\ part_of(r123, cs_building). \\ in(X, Y) \leftarrow part_of(Z, Y) \wedge in(X, Z). \end{cases}$$

Query	Answer
?part_of(r123, B).	<i>part_of(r123, cs_building)</i>
?part_of(r023, cs_building).	

Example Queries

$$KB = \begin{cases} in(kim, r123). \\ part_of(r123, cs_building). \\ in(X, Y) \leftarrow part_of(Z, Y) \wedge in(X, Z). \end{cases}$$

Query	Answer
?part_of(r123, B).	part_of(r123, cs_building)
?part_of(r023, cs_building).	no
?in(kim, r023).	

Example Queries

$$KB = \begin{cases} in(kim, r123). \\ part_of(r123, cs_building). \\ in(X, Y) \leftarrow part_of(Z, Y) \wedge in(X, Z). \end{cases}$$

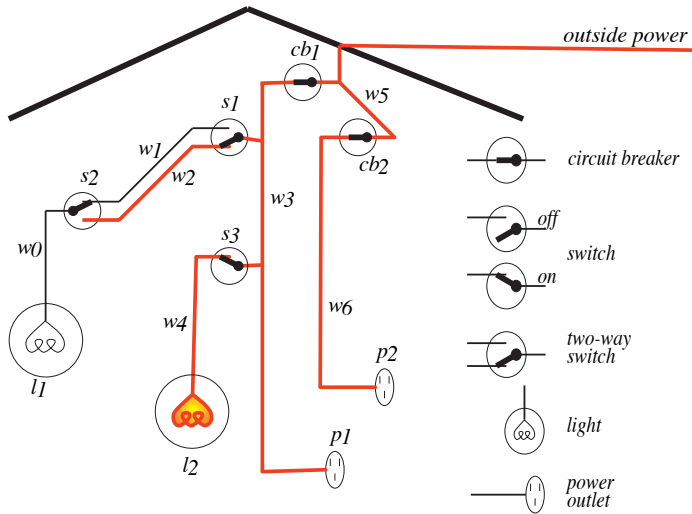
Query	Answer
?part_of(r123, B).	<i>part_of(r123, cs_building)</i>
?part_of(r023, cs_building).	<i>no</i>
?in(kim, r023).	<i>no</i>
?in(kim, B).	

Example Queries

$$KB = \begin{cases} in(kim, r123). \\ part_of(r123, cs_building). \\ in(X, Y) \leftarrow part_of(Z, Y) \wedge in(X, Z). \end{cases}$$

Query	Answer
?part_of(r123, B).	<i>part_of(r123, cs_building)</i>
?part_of(r023, cs_building).	<i>no</i>
?in(kim, r023).	<i>no</i>
?in(kim, B).	<i>in(kim, r123)</i> <i>in(kim, cs_building)</i>

Electrical Environment



Axiomatizing the Electrical Environment

% *light(L)* is true if *L* is a light

light(l₁). *light(l₂)*.

% *down(S)* is true if switch *S* is down

down(s₁). *up(s₂)*. *up(s₃)*.

% *ok(D)* is true if *D* is not broken

ok(l₁). *ok(l₂)*. *ok(cb₁)*. *ok(cb₂)*.

?*light(l₁)*. \implies

Axiomatizing the Electrical Environment

% *light(L)* is true if *L* is a light

light(l₁). *light(l₂)*.

% *down(S)* is true if switch *S* is down

down(s₁). *up(s₂)*. *up(s₃)*.

% *ok(D)* is true if *D* is not broken

ok(l₁). *ok(l₂)*. *ok(cb₁)*. *ok(cb₂)*.

?*light(l₁)*. \Rightarrow *yes*

?*light(l₆)*. \Rightarrow

Axiomatizing the Electrical Environment

% *light(L)* is true if *L* is a light

light(l₁). *light(l₂)*.

% *down(S)* is true if switch *S* is down

down(s₁). *up(s₂)*. *up(s₃)*.

% *ok(D)* is true if *D* is not broken

ok(l₁). *ok(l₂)*. *ok(cb₁)*. *ok(cb₂)*.

?*light(l₁)*. \implies *yes*

?*light(l₆)*. \implies *no*

?*up(X)*. \implies

Axiomatizing the Electrical Environment

% *light(L)* is true if *L* is a light

light(l₁). *light(l₂)*.

% *down(S)* is true if switch *S* is down

down(s₁). *up(s₂)*. *up(s₃)*.

% *ok(D)* is true if *D* is not broken

ok(l₁). *ok(l₂)*. *ok(cb₁)*. *ok(cb₂)*.

?*light(l₁)*. \implies *yes*

?*light(l₆)*. \implies *no*

?*up(X)*. \implies *up(s₂)*, *up(s₃)*

connected_to(X, Y) is true if component X is connected to Y

connected_to(w_0, w_1) \leftarrow *up*(s_2).

connected_to(w_0, w_2) \leftarrow *down*(s_2).

connected_to(w_1, w_3) \leftarrow *up*(s_1).

connected_to(w_2, w_3) \leftarrow *down*(s_1).

connected_to(w_4, w_3) \leftarrow *up*(s_3).

connected_to(p_1, w_3).

?*connected_to*(w_0, W). \implies

connected_to(X, Y) is true if component X is connected to Y

connected_to(w_0, w_1) \leftarrow *up*(s_2).

connected_to(w_0, w_2) \leftarrow *down*(s_2).

connected_to(w_1, w_3) \leftarrow *up*(s_1).

connected_to(w_2, w_3) \leftarrow *down*(s_1).

connected_to(w_4, w_3) \leftarrow *up*(s_3).

connected_to(p_1, w_3).

?*connected_to*(w_0, W). \implies $W = w_1$

?*connected_to*(w_1, W). \implies

connected_to(X, Y) is true if component X is connected to Y

connected_to(w_0, w_1) \leftarrow *up*(s_2).

connected_to(w_0, w_2) \leftarrow *down*(s_2).

connected_to(w_1, w_3) \leftarrow *up*(s_1).

connected_to(w_2, w_3) \leftarrow *down*(s_1).

connected_to(w_4, w_3) \leftarrow *up*(s_3).

connected_to(p_1, w_3).

?*connected_to*(w_0, W). \implies $W = w_1$

?*connected_to*(w_1, W). \implies *no*

?*connected_to*(Y, w_3). \implies

connected_to(X, Y) is true if component X is connected to Y

connected_to(w_0, w_1) \leftarrow *up*(s_2).

connected_to(w_0, w_2) \leftarrow *down*(s_2).

connected_to(w_1, w_3) \leftarrow *up*(s_1).

connected_to(w_2, w_3) \leftarrow *down*(s_1).

connected_to(w_4, w_3) \leftarrow *up*(s_3).

connected_to(p_1, w_3).

?*connected_to*(w_0, W). \implies $W = w_1$

?*connected_to*(w_1, W). \implies *no*

?*connected_to*(Y, w_3). \implies $Y = w_2, Y = w_4, Y = p_1$

?*connected_to*(X, W). \implies

connected_to(X, Y) is true if component X is connected to Y

connected_to(w_0, w_1) \leftarrow *up*(s_2).

connected_to(w_0, w_2) \leftarrow *down*(s_2).

connected_to(w_1, w_3) \leftarrow *up*(s_1).

connected_to(w_2, w_3) \leftarrow *down*(s_1).

connected_to(w_4, w_3) \leftarrow *up*(s_3).

connected_to(p_1, w_3).

?*connected_to*(w_0, W). \implies $W = w_1$

?*connected_to*(w_1, W). \implies *no*

?*connected_to*(Y, w_3). \implies $Y = w_2, Y = w_4, Y = p_1$

?*connected_to*(X, W). \implies $X = w_0, W = w_1, \dots$

% *lit(L)* is true if the light *L* is lit

$lit(L) \leftarrow light(L) \wedge ok(L) \wedge live(L).$

% *live(C)* is true if there is power coming into *C*

$live(Y) \leftarrow$

$connected_to(Y, Z) \wedge$

$live(Z).$

$live(outside).$

This is a **recursive definition** of *live*.

Recursion and Mathematical Induction

$above(X, Y) \leftarrow on(X, Y).$

$above(X, Y) \leftarrow on(X, Z) \wedge above(Z, Y).$

This can be seen as:

- Recursive definition of *above*: prove *above* in terms of a base case (*on*) or a simpler instance of itself; or
- Way to prove *above* by mathematical induction: the base case is when there are no blocks between *X* and *Y*, and if you can prove *above* when there are n blocks between them, you can prove it when there are $n + 1$ blocks.

- Suppose you had a database using the relation:

enrolled(S, C)

which is true when student S is enrolled in course C .

- Can you define the relation:

empty_course(C)

which is true when course C has no students enrolled in it?

- Why? or Why not?

- Suppose you had a database using the relation:

enrolled(S, C)

which is true when student S is enrolled in course C .

- Can you define the relation:

empty_course(C)

which is true when course C has no students enrolled in it?

- Why? or Why not?

empty_course(C) doesn't logically follow from a set of *enrolled* relation because there are always models where someone is enrolled in a course!

- An **instance** of an atom or a clause is obtained by uniformly substituting terms for variables.
- A **substitution** is a finite set of the form $\{V_1/t_1, \dots, V_n/t_n\}$, where each V_i is a distinct variable and each t_i is a term.
- The **application** of a substitution $\sigma = \{V_1/t_1, \dots, V_n/t_n\}$ to an atom or clause e , written $e\sigma$, is the instance of e with every occurrence of V_i replaced by t_i .

Application Examples

The following are substitutions:

$$\sigma_1 = \{X/A, Y/b, Z/C, D/e\}$$

$$\sigma_2 = \{A/X, Y/b, C/Z, D/e\}$$

$$\sigma_3 = \{A/V, X/V, Y/b, C/W, Z/W, D/e\}$$

The following shows some applications:

$$p(A, b, C, D)\sigma_1 =$$

$$p(X, Y, Z, e)\sigma_1 =$$

$$p(A, b, C, D)\sigma_2 =$$

$$p(X, Y, Z, e)\sigma_2 =$$

$$p(A, b, C, D)\sigma_3 =$$

$$p(X, Y, Z, e)\sigma_3 =$$

Application Examples

The following are substitutions:

$$\sigma_1 = \{X/A, Y/b, Z/C, D/e\}$$

$$\sigma_2 = \{A/X, Y/b, C/Z, D/e\}$$

$$\sigma_3 = \{A/V, X/V, Y/b, C/W, Z/W, D/e\}$$

The following shows some applications:

$$p(A, b, C, D)\sigma_1 = p(A, b, C, e)$$

$$p(X, Y, Z, e)\sigma_1 =$$

$$p(A, b, C, D)\sigma_2 =$$

$$p(X, Y, Z, e)\sigma_2 =$$

$$p(A, b, C, D)\sigma_3 =$$

$$p(X, Y, Z, e)\sigma_3 =$$

Application Examples

The following are substitutions:

$$\sigma_1 = \{X/A, Y/b, Z/C, D/e\}$$

$$\sigma_2 = \{A/X, Y/b, C/Z, D/e\}$$

$$\sigma_3 = \{A/V, X/V, Y/b, C/W, Z/W, D/e\}$$

The following shows some applications:

$$p(A, b, C, D)\sigma_1 = p(A, b, C, e)$$

$$p(X, Y, Z, e)\sigma_1 = p(A, b, C, e)$$

$$p(A, b, C, D)\sigma_2 =$$

$$p(X, Y, Z, e)\sigma_2 =$$

$$p(A, b, C, D)\sigma_3 =$$

$$p(X, Y, Z, e)\sigma_3 =$$

Application Examples

The following are substitutions:

$$\sigma_1 = \{X/A, Y/b, Z/C, D/e\}$$

$$\sigma_2 = \{A/X, Y/b, C/Z, D/e\}$$

$$\sigma_3 = \{A/V, X/V, Y/b, C/W, Z/W, D/e\}$$

The following shows some applications:

$$p(A, b, C, D)\sigma_1 = p(A, b, C, e)$$

$$p(X, Y, Z, e)\sigma_1 = p(A, b, C, e)$$

$$p(A, b, C, D)\sigma_2 = p(X, b, Z, e)$$

$$p(X, Y, Z, e)\sigma_2 =$$

$$p(A, b, C, D)\sigma_3 =$$

$$p(X, Y, Z, e)\sigma_3 =$$

Application Examples

The following are substitutions:

$$\sigma_1 = \{X/A, Y/b, Z/C, D/e\}$$

$$\sigma_2 = \{A/X, Y/b, C/Z, D/e\}$$

$$\sigma_3 = \{A/V, X/V, Y/b, C/W, Z/W, D/e\}$$

The following shows some applications:

$$p(A, b, C, D)\sigma_1 = p(A, b, C, e)$$

$$p(X, Y, Z, e)\sigma_1 = p(A, b, C, e)$$

$$p(A, b, C, D)\sigma_2 = p(X, b, Z, e)$$

$$p(X, Y, Z, e)\sigma_2 = p(X, b, Z, e)$$

$$p(A, b, C, D)\sigma_3 =$$

$$p(X, Y, Z, e)\sigma_3 =$$

The following are substitutions:

$$\sigma_1 = \{X/A, Y/b, Z/C, D/e\}$$

$$\sigma_2 = \{A/X, Y/b, C/Z, D/e\}$$

$$\sigma_3 = \{A/V, X/V, Y/b, C/W, Z/W, D/e\}$$

The following shows some applications:

$$p(A, b, C, D)\sigma_1 = p(A, b, C, e)$$

$$p(X, Y, Z, e)\sigma_1 = p(A, b, C, e)$$

$$p(A, b, C, D)\sigma_2 = p(X, b, Z, e)$$

$$p(X, Y, Z, e)\sigma_2 = p(X, b, Z, e)$$

$$p(A, b, C, D)\sigma_3 = p(V, b, W, e)$$

$$p(X, Y, Z, e)\sigma_3 =$$

Application Examples

The following are substitutions:

$$\sigma_1 = \{X/A, Y/b, Z/C, D/e\}$$

$$\sigma_2 = \{A/X, Y/b, C/Z, D/e\}$$

$$\sigma_3 = \{A/V, X/V, Y/b, C/W, Z/W, D/e\}$$

The following shows some applications:

$$p(A, b, C, D)\sigma_1 = p(A, b, C, e)$$

$$p(X, Y, Z, e)\sigma_1 = p(A, b, C, e)$$

$$p(A, b, C, D)\sigma_2 = p(X, b, Z, e)$$

$$p(X, Y, Z, e)\sigma_2 = p(X, b, Z, e)$$

$$p(A, b, C, D)\sigma_3 = p(V, b, W, e)$$

$$p(X, Y, Z, e)\sigma_3 = p(V, b, W, e)$$

- Substitution σ is a **unifier** of e_1 and e_2 if $e_1\sigma = e_2\sigma$.
- Substitution σ is a **most general unifier** (mgu) of e_1 and e_2 if
 - ▶ σ is a unifier of e_1 and e_2 ; and
 - ▶ if substitution σ' also unifies e_1 and e_2 , then $e\sigma'$ is an instance of $e\sigma$ for all atoms e .
- If two atoms have a unifier, they have a most general unifier.

Unification Example

Which of the following are unifiers of $p(A, b, C, D)$ and $p(X, Y, Z, e)$:

$$\sigma_1 = \{X/A, Y/b, Z/C, D/e\}$$

$$\sigma_2 = \{Y/b, D/e\}$$

$$\sigma_3 = \{X/A, Y/b, Z/C, D/e, W/a\}$$

$$\sigma_4 = \{A/X, Y/b, C/Z, D/e\}$$

$$\sigma_5 = \{X/a, Y/b, Z/c, D/e\}$$

$$\sigma_6 = \{A/a, X/a, Y/b, C/c, Z/c, D/e\}$$

$$\sigma_7 = \{A/V, X/V, Y/b, C/W, Z/W, D/e\}$$

$$\sigma_8 = \{X/A, Y/b, Z/A, C/A, D/e\}$$

Which are most general unifiers?

Unification Example

$p(A, b, C, D)$ and $p(X, Y, Z, e)$ have as unifiers:

$$\sigma_1 = \{X/A, Y/b, Z/C, D/e\}$$

$$\sigma_4 = \{A/X, Y/b, C/Z, D/e\}$$

$$\sigma_7 = \{A/V, X/V, Y/b, C/W, Z/W, D/e\}$$

$$\sigma_6 = \{A/a, X/a, Y/b, C/c, Z/c, D/e\}$$

$$\sigma_8 = \{X/A, Y/b, Z/A, C/A, D/e\}$$

$$\sigma_3 = \{X/A, Y/b, Z/C, D/e, W/a\}$$

The first three are most general unifiers.

The following substitutions are not unifiers:

$$\sigma_2 = \{Y/b, D/e\}$$

$$\sigma_5 = \{X/a, Y/b, Z/c, D/e\}$$

```

1: procedure unify( $t_1, t_2$ )      ▷ Returns mgu of  $t_1$  and  $t_2$  or  $\perp$ .
2:    $E \leftarrow \{t_1 = t_2\}$       ▷ Set of equality statements
3:    $S \leftarrow \{\}$                 ▷ Substitution
4:   while  $E \neq \{\}$  do
5:     select and remove  $x = y$  from  $E$ 
6:     if  $y$  is not identical to  $x$  then
7:       if  $x$  is a variable then
8:         replace  $x$  with  $y$  in  $E$  and  $S$ 
9:          $S \leftarrow \{x/y\} \cup S$ 
10:      else if  $y$  is a variable then
11:        replace  $y$  with  $x$  in  $E$  and  $S$ 
12:         $S \leftarrow \{y/x\} \cup S$ 
13:      else if  $x$  is  $p(x_1, \dots, x_n)$  and  $y$  is
14:         $p(y_1, \dots, y_n)$  then
15:           $E \leftarrow E \cup \{x_1 = y_1, \dots, x_n = y_n\}$ 
16:        else
17:          return  $\perp$  ▷  $t_1$  and  $t_2$  do not unify
18:        return  $S$                 ▷  $S$  is mgu of  $t_1$  and  $t_2$ 

```

Atom g is a logical consequence of KB if and only if:

- g is an instance of a fact in KB , or
- there is an instance of a rule

$$g \leftarrow b_1 \wedge \dots \wedge b_k$$

in KB such that each b_i is a logical consequence of KB .

Aside: Debugging false conclusions

To debug answer g that is false in the intended interpretation:

- If g is a fact in KB , this fact is wrong.
- Otherwise, suppose g was proved using the rule:

$$g \leftarrow b_1 \wedge \dots \wedge b_k$$

where each b_i is a logical consequence of KB .

- ▶ If each b_i is true in the intended interpretation, this clause is false in the intended interpretation.
- ▶ If some b_i is false in the intended interpretation, debug b_i .

- A **proof** is a mechanically derivable demonstration that a formula logically follows from a knowledge base.
- Given a proof procedure, $KB \vdash g$ means g can be derived from knowledge base KB .
- Recall $KB \models g$ means g is true in all models of KB .
- A proof procedure is **sound** if $KB \vdash g$ implies $KB \models g$.
- A proof procedure is **complete** if $KB \models g$ implies $KB \vdash g$.

Bottom-up proof procedure

$KB \vdash g$ if there is g' added to C in this procedure where $g = g'\theta$:

$C := \{\}$;

repeat

select clause " $h \leftarrow b_1 \wedge \dots \wedge b_m$ " in KB such that
there is a substitution θ such that
for all i , there exists $b'_i \in C$ and θ'_i where $b_i\theta = b'_i\theta'_i$ and
there is no $h' \in C$ and θ' such that $h'\theta' = h\theta$

$C := C \cup \{h\theta\}$

until no more clauses can be selected.

Example

$live(Y) \leftarrow connected_to(Y, Z) \wedge live(Z).$ $live(outside).$
 $connected_to(w_6, w_5).$ $connected_to(w_5, outside).$

Example

$live(Y) \leftarrow connected_to(Y, Z) \wedge live(Z). \quad live(outside).$
 $connected_to(w_6, w_5). \quad connected_to(w_5, outside).$

$C = \{live(outside),$
 $\quad connected_to(w_6, w_5),$
 $\quad connected_to(w_5, outside),$
 $\quad live(w_5),$
 $\quad live(w_6)\}$

Soundness of bottom-up proof procedure

If $KB \vdash g$ then $KB \models g$.

- Suppose there is a g such that $KB \vdash g$ and $KB \not\models g$.
- Then there must be a first atom added to C that has an instance that isn't true in every model of KB . Call it h .

Soundness of bottom-up proof procedure

If $KB \vdash g$ then $KB \models g$.

- Suppose there is a g such that $KB \vdash g$ and $KB \not\models g$.
- Then there must be a first atom added to C that has an instance that isn't true in every model of KB . Call it h .
- Suppose h isn't true in model I of KB .
- There must be an instance of clause in KB of form

$$h' \leftarrow b_1 \wedge \dots \wedge b_m$$

where

Soundness of bottom-up proof procedure

If $KB \vdash g$ then $KB \models g$.

- Suppose there is a g such that $KB \vdash g$ and $KB \not\models g$.
- Then there must be a first atom added to C that has an instance that isn't true in every model of KB . Call it h .
- Suppose h isn't true in model I of KB .
- There must be an instance of clause in KB of form

$$h' \leftarrow b_1 \wedge \dots \wedge b_m$$

where $h = h'\theta$ and $b_i\theta$ is an instance of an element of C .

- ▶ Each $b_i\theta$ is true in I .
- ▶ h is false in I .
- ▶ So an instance of this clause is false in I .
- ▶ Therefore I isn't a model of KB .
- ▶ Contradiction.

Fixed Point

- The C generated by the bottom-up algorithm is called a **fixed point**.
- C can be infinite; we require the selection to be fair.
- **Herbrand interpretation**: The domain is the set of constants. We invent a constant if the KB or query doesn't contain one. Each constant denotes itself.

- The C generated by the bottom-up algorithm is called a **fixed point**.
- C can be infinite; we require the selection to be fair.
- **Herbrand interpretation**: The domain is the set of constants. We invent a constant if the KB or query doesn't contain one. Each constant denotes itself.
- Let I be the Herbrand interpretation in which every ground instance of every element of the fixed point is true and every other atom is false.

- The C generated by the bottom-up algorithm is called a **fixed point**.
- C can be infinite; we require the selection to be fair.
- **Herbrand interpretation**: The domain is the set of constants. We invent a constant if the KB or query doesn't contain one. Each constant denotes itself.
- Let I be the Herbrand interpretation in which every ground instance of every element of the fixed point is true and every other atom is false.
- I is a model of KB .
Proof:

Fixed Point

- The C generated by the bottom-up algorithm is called a **fixed point**.
- C can be infinite; we require the selection to be fair.
- **Herbrand interpretation**: The domain is the set of constants. We invent a constant if the KB or query doesn't contain one. Each constant denotes itself.
- Let I be the Herbrand interpretation in which every ground instance of every element of the fixed point is true and every other atom is false.
- I is a model of KB .
Proof: suppose $h \leftarrow b_1 \wedge \dots \wedge b_m$ in KB is false in I . Then h is false and each b_i is true in I . Thus h can be added to C . Contradiction to C being the fixed point.
- I is called a **Minimal Model**.

If $KB \models g$ then $KB \vdash g$.

- Suppose $KB \models g$. Then g is true in all models of KB .

If $KB \models g$ then $KB \vdash g$.

- Suppose $KB \models g$. Then g is true in all models of KB .
- Thus g is true in the minimal model.

If $KB \models g$ then $KB \vdash g$.

- Suppose $KB \models g$. Then g is true in all models of KB .
- Thus g is true in the minimal model.
- Thus g is in the fixed point.

If $KB \models g$ then $KB \vdash g$.

- Suppose $KB \models g$. Then g is true in all models of KB .
- Thus g is true in the minimal model.
- Thus g is in the fixed point.
- Thus g is generated by the bottom up algorithm.

If $KB \models g$ then $KB \vdash g$.

- Suppose $KB \models g$. Then g is true in all models of KB .
- Thus g is true in the minimal model.
- Thus g is in the fixed point.
- Thus g is generated by the bottom up algorithm.
- Thus $KB \vdash g$.

- A **generalized answer clause** is of the form

$$\text{yes}(t_1, \dots, t_k) \leftarrow a_1 \wedge a_2 \wedge \dots \wedge a_m,$$

where t_1, \dots, t_k are terms and a_1, \dots, a_m are atoms.

Top-down Proof procedure

- A **generalized answer clause** is of the form

$$\text{yes}(t_1, \dots, t_k) \leftarrow a_1 \wedge a_2 \wedge \dots \wedge a_m,$$

where t_1, \dots, t_k are terms and a_1, \dots, a_m are atoms.

- The **SLD resolution** of this generalized answer clause on a_i with the clause

$$a \leftarrow b_1 \wedge \dots \wedge b_p,$$

where a_i and a have most general unifier θ , is

$$\begin{aligned} &(\text{yes}(t_1, \dots, t_k) \leftarrow \\ & a_1 \wedge \dots \wedge a_{i-1} \wedge b_1 \wedge \dots \wedge b_p \wedge a_{i+1} \wedge \dots \wedge a_m) \theta. \end{aligned}$$

Top-down Proof Procedure

To solve query $?B$ with variables V_1, \dots, V_k :

Set ac to generalized answer clause $yes(V_1, \dots, V_k) \leftarrow B$

while ac is not an answer **do**

 Suppose ac is $yes(t_1, \dots, t_k) \leftarrow a_1 \wedge a_2 \wedge \dots \wedge a_m$

select atom a_i in the body of ac

choose clause $a \leftarrow b_1 \wedge \dots \wedge b_p$ in KB

 Rename all variables in $a \leftarrow b_1 \wedge \dots \wedge b_p$

 Let θ be the most general unifier of a_i and a .

 Fail if they don't unify

 Set ac to $(yes(t_1, \dots, t_k) \leftarrow a_1 \wedge \dots \wedge a_{i-1} \wedge$
 $b_1 \wedge \dots \wedge b_p \wedge a_{i+1} \wedge \dots \wedge a_m)\theta$

end while.

Answer is $V_1 = t_1, \dots, V_k = t_k$

where ac is $yes(t_1, \dots, t_k) \leftarrow$

Example

$live(Y) \leftarrow connected_to(Y, Z) \wedge live(Z). \quad live(outside).$
 $connected_to(w_6, w_5). \quad connected_to(w_5, outside).$
 $?live(A).$

Example

$live(Y) \leftarrow connected_to(Y, Z) \wedge live(Z). \quad live(outside).$

$connected_to(w_6, w_5). \quad connected_to(w_5, outside).$

$?live(A).$

$yes(A) \leftarrow live(A).$

$yes(A) \leftarrow connected_to(A, Z_1) \wedge live(Z_1).$

$yes(w_6) \leftarrow live(w_5).$

$yes(w_6) \leftarrow connected_to(w_5, Z_2) \wedge live(Z_2).$

$yes(w_6) \leftarrow live(outside).$

$yes(w_6) \leftarrow .$

Function Symbols

- Often we want to refer to individuals in terms of components.
- Examples: 4:55 p.m. English sentences. A classlist.
- We extend the notion of **term**. So that a term can be $f(t_1, \dots, t_n)$ where f is a **function symbol** and the t_i are terms.
- In an interpretation and with a variable assignment, term $f(t_1, \dots, t_n)$ denotes an individual in the domain.
- One function symbol and one constant can refer to infinitely many individuals.

- A list is an ordered sequence of elements.
- Let's use the constant *nil* to denote the empty list, and the function *cons(H, T)* to denote the list with first element *H* and rest-of-list *T*. **These are not built-in.**
- The list containing *sue*, *kim* and *randy* is

cons(sue, cons(kim, cons(randy, nil)))

- *append(X, Y, Z)* is true if list *Z* contains the elements of *X* followed by the elements of *Y*

append(nil, Z, Z).

append(cons(A, X), Y, cons(A, Z)) ← append(X, Y, Z).

Unification with function symbols

- Consider a knowledge base consisting of one fact:

$It(X, s(X)).$

- Should the following query succeed?

ask $It(Y, Y).$

Unification with function symbols

- Consider a knowledge base consisting of one fact:

$It(X, s(X)).$

- Should the following query succeed?

ask $It(Y, Y).$

- What does the top-down proof procedure give?

Unification with function symbols

- Consider a knowledge base consisting of one fact:

$It(X, s(X)).$

- Should the following query succeed?

ask $It(Y, Y).$

- What does the top-down proof procedure give?
- Solution: variable X should not unify with a term that contains X inside.
E.g., X should not unify with $s(X)$.
Simple modification of the unification algorithm, which Prolog does not do!

Complete Knowledge Assumption

- Often you want to assume that your knowledge is complete.
- **Example:** assume that a database of what students are enrolled in a course is complete. We don't want to have to state all negative enrolment facts!
- The definite clause language is **monotonic**: adding clauses can't invalidate a previous conclusion.
- Under the complete knowledge assumption, the system is **non-monotonic**: adding clauses can invalidate a previous conclusion.

Equality

Equality is a special predicate symbol with a standard domain-independent intended interpretation.

- Suppose interpretation $I = \langle D, \phi, \pi \rangle$.
- t_1 and t_2 are ground terms then $t_1 = t_2$ is true in interpretation I if t_1 and t_2 denote the same individual. That is, $t_1 = t_2$ if $\phi(t_1)$ is the same as $\phi(t_2)$.
- $t_1 \neq t_2$ when t_1 and t_2 denote different individuals.

Equality

Equality is a special predicate symbol with a standard domain-independent intended interpretation.

- Suppose interpretation $I = \langle D, \phi, \pi \rangle$.
- t_1 and t_2 are ground terms then $t_1 = t_2$ is true in interpretation I if t_1 and t_2 denote the same individual. That is, $t_1 = t_2$ if $\phi(t_1)$ is the same as $\phi(t_2)$.
- $t_1 \neq t_2$ when t_1 and t_2 denote different individuals.
- Example:

$D = \{ \text{✂}, \text{☎}, \text{✎} \}$.

$\phi(\text{phone}) = \text{☎}$, $\phi(\text{pencil}) = \text{✎}$, $\phi(\text{telephone}) = \text{☎}$

What equalities and inequalities hold?

Equality is a special predicate symbol with a standard domain-independent intended interpretation.

- Suppose interpretation $I = \langle D, \phi, \pi \rangle$.
- t_1 and t_2 are ground terms then $t_1 = t_2$ is true in interpretation I if t_1 and t_2 denote the same individual. That is, $t_1 = t_2$ if $\phi(t_1)$ is the same as $\phi(t_2)$.
- $t_1 \neq t_2$ when t_1 and t_2 denote different individuals.
- Example:

$D = \{ \text{✂}, \text{☎}, \text{✎} \}$.

$\phi(\text{phone}) = \text{☎}$, $\phi(\text{pencil}) = \text{✎}$, $\phi(\text{telephone}) = \text{☎}$

What equalities and inequalities hold?

$\text{phone} = \text{telephone}$, $\text{phone} = \text{phone}$, $\text{pencil} = \text{pencil}$,

$\text{telephone} = \text{telephone}$

$\text{pencil} \neq \text{phone}$, $\text{pencil} \neq \text{telephone}$

Equality

Equality is a special predicate symbol with a standard domain-independent intended interpretation.

- Suppose interpretation $I = \langle D, \phi, \pi \rangle$.
- t_1 and t_2 are ground terms then $t_1 = t_2$ is true in interpretation I if t_1 and t_2 denote the same individual. That is, $t_1 = t_2$ if $\phi(t_1)$ is the same as $\phi(t_2)$.
- $t_1 \neq t_2$ when t_1 and t_2 denote different individuals.

- Example:

$D = \{ \text{✂}, \text{☎}, \text{✎} \}$.

$\phi(\text{phone}) = \text{☎}$, $\phi(\text{pencil}) = \text{✎}$, $\phi(\text{telephone}) = \text{☎}$

What equalities and inequalities hold?

$\text{phone} = \text{telephone}$, $\text{phone} = \text{phone}$, $\text{pencil} = \text{pencil}$,

$\text{telephone} = \text{telephone}$

$\text{pencil} \neq \text{phone}$, $\text{pencil} \neq \text{telephone}$

- Equality does not mean similarity!

Properties of Equality

Equality is:

- Reflexive: $X = X$
- Symmetric: if $X = Y$ then $Y = X$
- Transitive: if $X = Y$ and $Y = Z$ then $X = Z$

For each n -ary function symbol f

$$f(X_1, \dots, X_n) = f(Y_1, \dots, Y_n) \text{ if } X_1 = Y_1 \text{ and } \dots \text{ and } X_n = Y_n.$$

For each n -ary predicate symbol p

$$p(X_1, \dots, X_n) \text{ if } p(Y_1, \dots, Y_n) \text{ and } X_1 = Y_1 \text{ and } \dots \text{ and } X_n = Y_n.$$

Unique Names Assumption

- Suppose the only clauses for *enrolled* are

enrolled(sam, cs222)

enrolled(chris, cs222)

enrolled(sam, cs873)

To conclude $\neg \textit{enrolled}(\textit{chris}, \textit{cs873})$, what do we need to assume?

Unique Names Assumption

- Suppose the only clauses for *enrolled* are

enrolled(sam, cs222)

enrolled(chris, cs222)

enrolled(sam, cs873)

To conclude $\neg \textit{enrolled}(\textit{chris}, \textit{cs873})$, what do we need to assume?

- ▶ All other enrolled facts are false
- ▶ Inequalities:

$$\textit{sam} \neq \textit{chris} \wedge \textit{cs873} \neq \textit{cs222}$$

- The **unique names assumption (UNA)** is the assumption that distinct ground terms denote different individuals.

Completion of a knowledge base: propositional case

- Suppose the rules for atom a are

$$a \leftarrow b_1.$$

\vdots

$$a \leftarrow b_n.$$

equivalently $a \leftarrow b_1 \vee \dots \vee b_n.$

- Under the Complete Knowledge Assumption, if a is true, one of the b_i must be true:

$$a \rightarrow b_1 \vee \dots \vee b_n.$$

- Thus, the clauses for a mean

$$a \leftrightarrow b_1 \vee \dots \vee b_n$$

Clark Normal Form

The **Clark normal form** of the clause

$$p(t_1, \dots, t_k) \leftarrow B.$$

is the clause

$$p(V_1, \dots, V_k) \leftarrow \exists W_1 \dots \exists W_m V_1 = t_1 \wedge \dots \wedge V_k = t_k \wedge B.$$

where

- V_1, \dots, V_k are k variables that did not appear in the original clause
- W_1, \dots, W_m are the original variables in the clause.

Clark Normal Form

The **Clark normal form** of the clause

$$p(t_1, \dots, t_k) \leftarrow B.$$

is the clause

$$p(V_1, \dots, V_k) \leftarrow \exists W_1 \dots \exists W_m V_1 = t_1 \wedge \dots \wedge V_k = t_k \wedge B.$$

where

- V_1, \dots, V_k are k variables that did not appear in the original clause
- W_1, \dots, W_m are the original variables in the clause.
- When the clause is an atomic clause, B is *true*.
- Often can be simplified by replacing $\exists W V = W \wedge p(W)$ with $P(V)$.

Clark normal form

For the clauses

$student(mary).$

$student(sam).$

$student(X) \leftarrow undergrad(X).$

the Clark normal form is

$student(V) \leftarrow V = mary.$

$student(V) \leftarrow V = sam.$

$student(V) \leftarrow \exists X V = X \wedge undergrad(X).$

Clark's Completion

Suppose all of the clauses for p are put into Clark normal form, with the same set of introduced variables, giving

$$p(V_1, \dots, V_k) \leftarrow B_1.$$

\vdots

$$p(V_1, \dots, V_k) \leftarrow B_n.$$

which is equivalent to

$$p(V_1, \dots, V_k) \leftarrow B_1 \vee \dots \vee B_n.$$

Clark's completion of predicate p is the equivalence

$$\forall V_1 \dots \forall V_k p(V_1, \dots, V_k) \leftrightarrow B_1 \vee \dots \vee B_n$$

If there are no clauses for p ,

Clark's Completion

Suppose all of the clauses for p are put into Clark normal form, with the same set of introduced variables, giving

$$p(V_1, \dots, V_k) \leftarrow B_1.$$

\vdots

$$p(V_1, \dots, V_k) \leftarrow B_n.$$

which is equivalent to

$$p(V_1, \dots, V_k) \leftarrow B_1 \vee \dots \vee B_n.$$

Clark's completion of predicate p is the equivalence

$$\forall V_1 \dots \forall V_k p(V_1, \dots, V_k) \leftrightarrow B_1 \vee \dots \vee B_n$$

If there are no clauses for p , the completion results in

$$\forall V_1 \dots \forall V_k p(V_1, \dots, V_k) \leftrightarrow \textit{false}$$

Clark's completion of a knowledge base consists of the completion of every predicate symbol along the unique names assumption.

Completion example

$p \leftarrow q \wedge \sim r.$

$p \leftarrow s.$

$q \leftarrow \sim s.$

$r \leftarrow \sim t.$

$t.$

$s \leftarrow w.$

Completion Example

Consider the recursive definition:

$passed_each([], St, MinPass).$

$passed_each([C|R], St, MinPass) \leftarrow$
 $passed(St, C, MinPass) \wedge$
 $passed_each(R, St, MinPass).$

In Clark normal form, this can be written as

Completion Example

Consider the recursive definition:

$$\begin{aligned} & \textit{passed_each}([], St, MinPass). \\ & \textit{passed_each}([C|R], St, MinPass) \leftarrow \\ & \quad \textit{passed}(St, C, MinPass) \wedge \\ & \quad \textit{passed_each}(R, St, MinPass). \end{aligned}$$

In Clark normal form, this can be written as

$$\begin{aligned} & \textit{passed_each}(L, S, M) \leftarrow L = []. \\ & \textit{passed_each}(L, S, M) \leftarrow \\ & \quad \exists C \exists R L = [C|R] \wedge \textit{passed}(S, C, M) \wedge \textit{passed_each}(R, S, M). \end{aligned}$$

Here we renamed the variables as appropriate. Thus, Clark's completion of *passed_each* is

Completion Example

Consider the recursive definition:

$$\begin{aligned} & \textit{passed_each}([], St, MinPass). \\ & \textit{passed_each}([C|R], St, MinPass) \leftarrow \\ & \quad \textit{passed}(St, C, MinPass) \wedge \\ & \quad \textit{passed_each}(R, St, MinPass). \end{aligned}$$

In Clark normal form, this can be written as

$$\begin{aligned} & \textit{passed_each}(L, S, M) \leftarrow L = []. \\ & \textit{passed_each}(L, S, M) \leftarrow \\ & \quad \exists C \exists R L = [C|R] \wedge \textit{passed}(S, C, M) \wedge \textit{passed_each}(R, S, M). \end{aligned}$$

Here we renamed the variables as appropriate. Thus, Clark's completion of *passed_each* is

$$\begin{aligned} & \forall L \forall S \forall M \textit{passed_each}(L, S, M) \leftrightarrow L = [] \vee \\ & \quad \exists C \exists R L = [C|R] \wedge \textit{passed}(S, C, M) \wedge \textit{passed_each}(R, S, M). \end{aligned}$$

Clark's Completion of a KB

- Clark's completion of a knowledge base consists of the completion of every predicate.
- The completion of an n -ary predicate p with no clauses is $p(V_1, \dots, V_n) \leftrightarrow \text{false}$.
- You can interpret negations in the body of clauses. $\sim a$ means a is false under the complete knowledge assumption. $\sim a$ is replaced by $\neg a$ in the completion. This is **negation as failure**.

Defining *empty_course*

Given database of:

- *course*(C) that is true if C is a course
- *enrolled*(S, C) that is true if student S is enrolled in course C .

Define *empty_course*(C) that is true if there are no students enrolled in course C .

Defining *empty_course*

Given database of:

- *course*(C) that is true if C is a course
- *enrolled*(S, C) that is true if student S is enrolled in course C .

Define *empty_course*(C) that is true if there are no students enrolled in course C .

- Using negation as failure, *empty_course*(C) can be defined by
$$\text{empty_course}(C) \leftarrow \text{course}(C) \wedge \sim \text{has_enrollment}(C).$$
$$\text{has_enrollment}(C) \leftarrow \text{enrolled}(S, C).$$

Defining *empty_course*

Given database of:

- *course*(C) that is true if C is a course
- *enrolled*(S, C) that is true if student S is enrolled in course C .

Define *empty_course*(C) that is true if there are no students enrolled in course C .

- Using negation as failure, *empty_course*(C) can be defined by
$$\begin{aligned} \text{empty_course}(C) &\leftarrow \text{course}(C) \wedge \sim \text{has_enrollment}(C). \\ \text{has_enrollment}(C) &\leftarrow \text{enrolled}(S, C). \end{aligned}$$
- The completion of this is:

Defining *empty_course*

Given database of:

- *course*(C) that is true if C is a course
- *enrolled*(S, C) that is true if student S is enrolled in course C .

Define *empty_course*(C) that is true if there are no students enrolled in course C .

- Using negation as failure, *empty_course*(C) can be defined by

$$empty_course(C) \leftarrow course(C) \wedge \sim has_enrollment(C).$$

$$has_enrollment(C) \leftarrow enrolled(S, C).$$

- The completion of this is:

$$\forall C \text{ empty_course}(C) \iff course(C) \wedge \neg has_enrollment(C).$$

$$\forall C \text{ has_enrollment}(C) \iff \exists S \text{ enrolled}(S, C).$$

Bottom-up negation as failure interpreter

$C := \{\}$;

repeat

 either

 select $r \in KB$ such that

r is " $h \leftarrow b_1 \wedge \dots \wedge b_m$ "

$b_i \in C$ for all i , and

$h \notin C$;

$C := C \cup \{h\}$

 or

 select h such that for every rule " $h \leftarrow b_1 \wedge \dots \wedge b_m$ " $\in KB$

 either for some $b_i, \sim b_i \in C$

 or some $b_i = \sim g$ and $g \in C$

$C := C \cup \{\sim h\}$

until no more selections are possible

Negation as failure example

$p \leftarrow q \wedge \sim r.$

$p \leftarrow s.$

$q \leftarrow \sim s.$

$r \leftarrow \sim t.$

$t.$

$s \leftarrow w.$

Top-Down negation as failure proof procedure

- If the proof for a fails, you can conclude $\sim a$.
- Failure can be defined recursively:
Suppose you have rules for atom a :

$$a \leftarrow b_1$$

\vdots

$$a \leftarrow b_n$$

If each body b_i fails, a fails.

- A body fails if one of the conjuncts in the body fails.
- Note that you need *finite* failure. Example $p \leftarrow p$.

$p(X) \leftarrow \sim q(X) \wedge r(X).$

$q(a).$

$q(b).$

$r(d).$

ask $p(X).$

- What is the answer to the query?

$p(X) \leftarrow \sim q(X) \wedge r(X).$

$q(a).$

$q(b).$

$r(d).$

ask $p(X).$

- What is the answer to the query?
- How can a top-down proof procedure find the answer?

$p(X) \leftarrow \sim q(X) \wedge r(X).$

$q(a).$

$q(b).$

$r(d).$

ask $p(X).$

- What is the answer to the query?
- How can a top-down proof procedure find the answer?
- Delay the subgoal until it is bound enough.
Sometimes it never gets bound enough — “floundering”.

Problematic Cases

$p(X) \leftarrow \sim q(X)$

$q(X) \leftarrow \sim r(X)$

$r(a)$

ask $p(X)$.

- What is the answer?

Problematic Cases

$p(X) \leftarrow \sim q(X)$

$q(X) \leftarrow \sim r(X)$

$r(a)$

ask $p(X)$.

- What is the answer?
- What does delaying do?

Problematic Cases

$p(X) \leftarrow \sim q(X)$

$q(X) \leftarrow \sim r(X)$

$r(a)$

ask $p(X)$.

- What is the answer?
- What does delaying do?
- How can this be implemented?

Natural Language Understanding

- We want to communicate with computers using natural language (spoken and written).
 - ▶ unstructured natural language — allow any statements, but make mistakes or failure.
 - ▶ controlled natural language — only allow unambiguous statements that can be interpreted (e.g., in supermarkets or for doctors).
- There is a vast amount of information in natural language.
- Understanding language to extract information or answering questions is more difficult than getting extracting gestalt properties such as topic, or choosing a help page.
- Many of the problems of AI are explicit in natural language understanding. “AI complete”.

- **Syntax** describes the form of language (using a grammar).
- **Semantics** provides the meaning of language.
- **Pragmatics** explains the purpose or the use of language (how utterances relate to the world).

Examples:

- *This lecture is about natural language.*
- *The green frogs sleep soundly.*
- *Colorless green ideas sleep furiously.*
- *Furiously sleep ideas green colorless.*

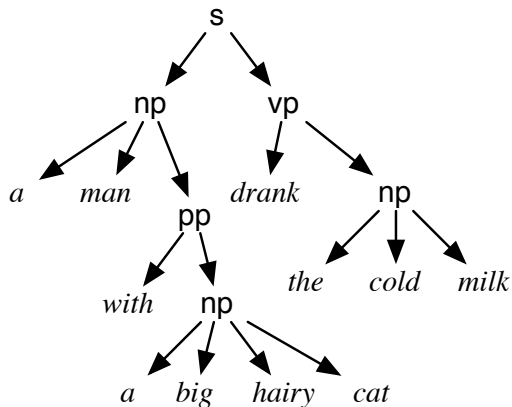
Beyond N-grams

- *A man with a big hairy cat drank the cold milk.*
- Who or what drank the milk?

Beyond N-grams

- *A man with a big hairy cat drank the cold milk.*
- Who or what drank the milk?

Simple syntax diagram:



Context-free grammar

- A **terminal symbol** is a word (perhaps including punctuation).
- A **non-terminal symbol** can be rewritten as a sequence of terminal and non-terminal symbols, e.g.,

$sentence \mapsto noun_phrase, verb_phrase$

$verb_phrase \mapsto verb, noun_phrase$

$verb \mapsto [drank]$

- Can be written as a logic program, where a sentence is a sequence of words:

$sentence(S) \leftarrow noun_phrase(N), verb_phrase(V), append(N, V, S).$

To say word “drank” is a verb:

$verb([drank]).$

Difference Lists

- Non-terminal symbol s becomes a predicate with two arguments, $s(T_1, T_2)$, meaning:
 - ▶ T_2 is an ending of the list T_1
 - ▶ all of the words in T_1 before T_2 form a sequence of words of the category s .
- Lists T_1 and T_2 together form a **difference list**.
- “the student” is a noun phrase:

noun_phrase([*the, student, passed, the, course*],
[*passed, the, course*])

Difference Lists

- Non-terminal symbol s becomes a predicate with two arguments, $s(T_1, T_2)$, meaning:
 - ▶ T_2 is an ending of the list T_1
 - ▶ all of the words in T_1 before T_2 form a sequence of words of the category s .
- Lists T_1 and T_2 together form a **difference list**.
- “the student” is a noun phrase:

noun_phrase([*the, student, passed, the, course*],
[*passed, the, course*])

- The word “drank” is a verb:

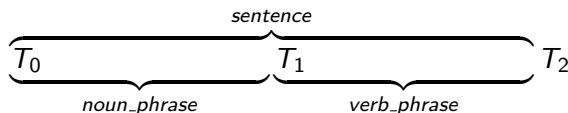
verb([*drank*| W], W).

Definite clause grammar

The grammar rule

$$\textit{sentence} \mapsto \textit{noun_phrase}, \textit{verb_phrase}$$

means that there is a sentence between T_0 and T_2 if there is a noun phrase between T_0 and T_1 and a verb phrase between T_1 and T_2 :

$$\begin{aligned} \textit{sentence}(T_0, T_2) \leftarrow \\ \textit{noun_phrase}(T_0, T_1) \wedge \\ \textit{verb_phrase}(T_1, T_2). \end{aligned}$$


Definite clause grammar rules

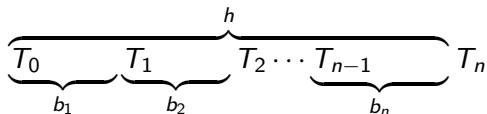
The rewriting rule

$$h \mapsto b_1, b_2, \dots, b_n$$

says that h is b_1 then b_2, \dots , then b_n :

$$\begin{aligned} h(T_0, T_n) \leftarrow & \\ & b_1(T_0, T_1) \wedge \\ & b_2(T_1, T_2) \wedge \\ & \vdots \\ & b_n(T_{n-1}, T_n). \end{aligned}$$

using the interpretation



Terminal Symbols

Non-terminal h gets mapped to the terminal symbols, t_1, \dots, t_n :

$$h([t_1, \dots, t_n | T], T)$$

using the interpretation

$$\overbrace{t_1, \dots, t_n}^h T$$

Thus, $h(T_1, T_2)$ is true if $T_1 = [t_1, \dots, t_n | T_2]$.

Complete Context Free Grammar Example

see

http://artint.info/code/Prolog/ch12/cfg_simple.pl

What will the following query return?

noun_phrase([the, student, passed, the, course, with, a, computer], R).

Complete Context Free Grammar Example

see

http://artint.info/code/Prolog/ch12/cfg_simple.pl

What will the following query return?

noun_phrase([the, student, passed, the, course, with, a, computer], R).

How many answers does the following query have?

sentence([the, student, passed, the, course, with, a, computer], R).

Two mechanisms can make the grammar more expressive:
extra arguments to the non-terminal symbols
arbitrary conditions on the rules.

We have a Turing-complete programming language at our disposal!

Add an extra argument representing a parse tree:

$$\begin{aligned} \textit{sentence}(T_0, T_2, s(NP, VP)) \leftarrow \\ \textit{noun_phrase}(T_0, T_1, NP) \wedge \\ \textit{verb_phrase}(T_1, T_2, VP). \end{aligned}$$

Enforcing Constraints

Add an argument representing the number (singular or plural), as well as the parse tree:

$$\begin{aligned} \textit{sentence}(T_0, T_2, \textit{Num}, s(\textit{NP}, \textit{VP})) \leftarrow \\ \textit{noun_phrase}(T_0, T_1, \textit{Num}, \textit{NP}) \wedge \\ \textit{verb_phrase}(T_1, T_2, \textit{Num}, \textit{VP}). \end{aligned}$$

The parse tree can return the determiner (definite or indefinite), number, modifiers (adjectives) and any prepositional phrase:

$$\begin{aligned} \textit{noun_phrase}(T, T, \textit{Num}, \textit{no_np}). \\ \textit{noun_phrase}(T_0, T_4, \textit{Num}, \textit{np}(\textit{Det}, \textit{Num}, \textit{Mods}, \textit{Noun}, \textit{PP})) \leftarrow \\ \textit{det}(T_0, T_1, \textit{Num}, \textit{Det}) \wedge \\ \textit{modifiers}(T_1, T_2, \textit{Mods}) \wedge \\ \textit{noun}(T_2, T_3, \textit{Num}, \textit{Noun}) \wedge \\ \textit{pp}(T_3, T_4, \textit{PP}). \end{aligned}$$

Complete Example

see

http://artint.info/code/Prolog/ch12/nl_numbera.pl

- How can we get from natural language to a query or to logical statements?
- Goal: map natural language to a query that can be asked of a knowledge base.
- Add arguments representing the individual and the relations about that individual. E.g.,

noun_phrase(T_0, T_1, O, C_0, C_1)

means

- ▶ $T_0 - T_1$ is a difference list forming a noun phrase.
- ▶ The noun phrase refers to the individual O .
- ▶ C_0 is list of previous relations.
- ▶ C_1 is C_0 together with the relations on individual O given by the noun phrase.

Example natural language to query

see

http://artint.info/code/Prolog/ch12/nl_interface.pl

The student took many courses. Two computer science courses and one mathematics course were particularly difficult. The mathematics course. . .

The student took many courses. Two computer science courses and one mathematics course were particularly difficult. The mathematics course. . .

*Who was the captain of the Titanic?
Was she tall?*